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MMATHEMATICAL MODELLING OF GAS NON-ORDINARY FLOW IN MAIN PIPELINES

Introduction

Analyses of a reliability of the main gas pipeline's exploitation has shown high probability of the main gas pipeline's some sections damage and gas leakage and as a result the gas pressure and expenditure alteration when non-stationary processes are in progress[1-3]. After some time gas leakage (under some conditions), it is possible establishment a new stationary state of gas movement in the pipelines has stationary character. That is way it is necessary to study as a non-stationary stage as well the stationary stage of gas movement in the pipelines having gas escape in the some sections of the main gas pipeline[1-2].

In this article we study only large-scale gas leakage problem from the main gas pipeline and we consider this question as a reverse task of hydraulic calculation problem.

Statement of the Problem

There are many scientific articles denoted to the problem of gas leakage problem from the main gas pipeline[4-9]. It is known analytical method of determination a large-scale gas escape location on the simple section of main gas pipeline [1], using data of the gas pressures and expenditure at the entrance and ending of the gas pipeline. But this method cannot be used for main gas pipelines with several sections and branches if previously would not be discovered the location of the section with gas escape. The method offered by us is devoid from this default.

So the problem can be formulated as follows: In the complex main gas pipeline with several branches and sections first of all the placement of the section having accidental gas escape is determined using minimal information (data of the gas pressures and expenditure at the main gas pipeline's entrance and ending points before and after gas escape) and then defined location of the accidental gas escape in the determined section of main pipeline.

Thus suppose that there is a complex main gas pipeline having $n-1$ off-shots, with expenses q_k $(k\overline{1,n-1})$ and the pipeline is divided by off-shots on n simple sections with length L_k $(k=\overline{1,n})$. If at the entrance of pipeline gas expanses in unit of time is M_0, then at the entrance of the per simple sections the gas expanses are calculated in the following way

$$M_1 = M_0 \ , \quad M_k = M_{k-1} - q_{k-1}, \qquad k = \overline{(2,n)},$$

where numbering is performed from the beginning of the pipeline to the ending.

As it is known in case of gas stationary movement in the horizontal gas pipeline exist the following equality [1]:

$$P_1^2 - P_2^2 = \sum_{k=1}^{n} M_k^2 \beta_k L_k, \qquad (1)$$

where $\beta_k = \dfrac{\lambda_k ZRT}{F_k^2 D_k}$, P1 and P2 are values of the pressures at the entrance and at the ending of the main gas pipe-line, respectively; Mk- are expenses of gas in the unit area of pipe-line for unit time in the branches; Lk- are lengths of simple section *k* of the main pipe-line; Z is a coefficient expressing deviation of natural gas from ideal gas; λ_k is a hydraulic resistance of a gas; T is an absolute temperature; R is a gas constant; Dk are diameters of pipelines; Fk are areas of branches profile sections.

Suppose that at the entrance of the main gas pipeline in the unit of time through pipe passes M_0 mass of gas, and at the ending of pipeline instant of gas mass Mn expenditure of gas is $Mn-Q$, which indicates that gas with mass Q is loosen, although the consumers (users) are getting the same mass of gas q_k $(k=\overline{1,n-1})$ which is conditioned by gas distributive stations (service management).

Let us suppose that gas leakage is placed on the section ***i*** and gas escape is located on the distance x $(0 \le x \le L_1)$ from the entrance of the section i. Also we suppose that accidental gas escape represents additional ramification of the main gas pipeline with expenditure Q. It is evidence that expenditure of gas is remained the same in the ramifications located before the section i but after the section i instead of expenditure M_k it will be $M_k - Q > 0$ $(k=\overline{1,n})$. In analogously of the right side of the equation (1) let us initiate the following functions $f_1(x)$:

$$f_1(x) = \sum_{k=1}^{n}[M_k - Q]^2 \beta_k L_k + Q[2M_1 - Q]\beta_1 x, \quad (0 < x \le L_1);$$

$$f_i(x) = \sum_{k=1}^{i-1} M_k^2 \beta_k L_k + \sum_{k=1}^{n} [M_k - Q]^2 \beta_k L_k + Q[2M_i - Q]\beta_i x,$$

$$i \in (2,3,\Lambda\ ,n-1). \qquad (0 < x \le L_1);$$

$$f_n(x) = \sum_{k=1}^{n-1} M_k^2 \beta_k L_k + [M_n - Q]^2 \beta_n L_n + Q[2M_n - Q]\beta_n x, \qquad (0 < x \le L_n).$$

Let us assume that after gas escape $\overline{P}_1^2$ and $\overline{P}_2^2$ are values of the gas pressures, at the entrance and ending of main pipeline, respectively (which are obtained by the measuring instruments).

Therefore, analogously of the equation (1) we have:

$$\overline{P}_1^2 - \overline{P}_2^2 = f_1(x). \qquad (2)$$

So for detection of the section of accidental gas escape and the point of gas escape in this section we have the following mathematical model (algorithm): first of all it is required to search such kind value i_0 from the sequence $i = \{1,2,\Lambda\ ,n\}$ and then the value of the x from the interval $\lfloor 0, I_{i_0} \rfloor$ which will satisfy the equation (2).

Theoretical investigation of the setting problem

For convenience here and further we are defining some properties of the above mentioned function $f_1(x)$:

1. Every function $f_1(x)$ $(i = \overline{1,n})$ represents linear increasing functions of x forasmuch as

$$Q[2M_1 - Q]\beta_1 > 0, \qquad (i = \overline{1,n})$$

2. The following equalities are correctness:

$$f_{i-1}(L_{i-1}) = f_i(0), \qquad (i = \overline{2,n}).$$

Indeed, let us consider the cases when $i = 1,2$ separately.

We will get:

$$f_1(L_1) = \sum_{k=1}^{n} (M_k - Q)^2 \beta_k L_k + Q[2M_1 - Q]\beta_1 L_1,$$

$$f_2(0) = M_1^2 \beta_1 L_1 + \sum_{k=1}^{n} (M_k - Q)^2 \beta_k L_k =$$

$$= \sum_{k=1}^{n} (M_k - Q)^2 \beta_k L_k + M_1^2 \beta_1 L_1 - (M_1 - Q)^2 \beta_1 L_1 =$$

$$= \sum_{k=1}^{n} (M_k - Q)^2 \beta_k L_k + Q(2M_1 - Q)\beta_1 L_1 = f_1(L_1)$$

When $i = 3,4,\Lambda\ ,n-1$ then

$$f_{i-1}(L_{i-1}) = \sum_{k=1}^{i-2} M_k^2 \beta_k L_k + \sum_{k=i-1}^{n} (M_k - Q)^2 \beta_k L_k + Q(2M_{i-1} - Q)\beta_{i-1} L_{i-1} =$$

$$= \sum_{k=1}^{i-1} M_k^2 \beta_k L_k + \sum_{k=1}^{n1} (M_k - Q)^2 \beta_k L_k - M_{i-1}^2 \beta_{i-1} L_{i-1} +$$

$$+ (M_{i-1} - Q)^2 \beta_{i-1} L_{i-1} + Q(2M_1 - Q)\beta_{i-1} L_{i-1} =$$

$$= \sum_{k=1}^{i-1} M_k^2 \beta_k L_k + \sum_{k=1}^{n1} (M_k - Q)^2 \beta_k L_k = f_1(0).$$

When $i = n$ we have

The last fully proofs proper 2.

Now arrange (on the axis) the segments with length **Li**, **i =(1,n)** step by step exactly in such a way that right tail-end point of the segment **i-1** and left tail-end point of the segment **i** will be coincide with each other.

Let us define function $f_i(x)$ on the each segment **i** in such a way, that beginning of the calculation for the argument ***x*** will be the left point of the segment **i** . In such a way arranged functions $f_i(x)$ represent continuous, sectional increasing linear functions.

From physical point of view above mentioned properties of the functions $f_1(x)$ means that the more is distance of the location of the accidental gas escape from the begging of the main gas pipeline, the bigger difference between the values of pressures' squares. Moreover this difference continuously defends on the distance in which the accidental gas escape is located from the begging point of the main gas pipeline. Using properties of the functions $f_i(x)$ it is possible to construct algorithm which gives possibility to find such kind values of **i0** and ***x*** which will satisfy equation (2).

For achievement of this aim first of all it is necessary to cheek up endings of branches. If for any value of **i0** from **i** the equality

$$f_{i_0}\left(L_{i_0}\right)=\overline{P}_1^2-\overline{P}_2^2 ,$$

is true, then gas accidental escape is located at the simple endings sections of the main pipeline. If the equality is not ful-filled for any value of **i**, then extracting the values of **i** from the sequence **i=1,2,..n**, it will be possible to find the least value of **i0** which will satisfy the following inequality

$$f_{i_0}\left(L_{i_0}\right)>\overline{P}_1^2-\overline{P}_2^2 .$$

In that case gas accidental escape is located within the section **i0**. If such kind inequality is not fulfilled for any values of **i0** from the sequence **i=1,2,..n-1**, then gas accidental escape is located on the last simple ending section numbered by **n**. In that case the following inequality will be true

$$f_n\left(L_n\right)\le\overline{P}_1^2-\overline{P}_2^2 , \text{ and } i_0=n .$$

Afterwards it is emplaced the location (number of the section **i0**) of the gas accidental escape the appropriate distance ***x*** can be defined by the solution of the following equation $f_{i_0}(x)=\overline{P}_1^2-\overline{P}_2^2$.

Namely if we have **i0=1** then

$$x=\left[\overline{P}_1^2-\overline{P}_2^2-\sum_{k=i-1}^{n}\left(M_k-Q\right)^2\beta_k L_k\right]/\left[Q\left(2M_1-Q\right)\beta_1\right]$$

If fulfilled the following inequality $2\le i_0\le n-1$ then

$$x=\left[\overline{P}_1^2-\overline{P}_2^2-\sum_{k=1}^{i_0-1}M_k^2\beta_k L_k-\sum_{k=1}^{n}\left(M_k-Q\right)^2\beta_k L_k\right]/\left[Q\left(2M_{i_0}-Q\right)\beta_{i_0}\right]$$

And at last if **i0=n**, then

$$x=\left[\overline{P}_1^2-\overline{P}_2^2-\sum_{k=1}^{i_0-1}M_k^2\beta_k L_k-\left(M_n-Q\right)^2\beta_k L_k\right]/\left[Q\left(2M_n-Q\right)\beta_n\right]$$

We have realized the algorithm. The calculations have been per-formed for the data taking from the several experiments. The results of calculations shown, that suggested model is available to define with high probability of the main gas pipeline's some sections damage and gas leakage and as a result the gas pressure and expenditure alteration when stationary process is in progress.

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ამ ნაშრომში განხილულია მათემატიკური მოდელი (ალგორითმი) რომელსაც შეუძლია რთული მაგისტრალური გაზსადენიდან განსაზღვროს ის მარტივი უბანი სადაც ადგილი აქვს გაზის ავარიულ გაჟონვას. Mმოცემული ალგორითმი არ მოითხოვს საწყისი ჰიდრავლიკური პარამეტრების ცოდნას ყოველი მარტივი უბნის სათავესა და ბოლოში (ამ ინფორმაციის მოპოვება ძალიან რთულია ტელემეტრიული საინფორმაციო სისტემების გარეშე). ალგორითმი დაფუძნებულია მათემატიკური მოდელზე რომელიც აღწერს გაზის სტაციონარულ დინებას რთულ მაგისტრალურ გაზსადენში, მის ანალიზურ ამოხსნებზე და ზოგიერთ რიცხვითი თვლის შედეცებზე.

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In this paper a mathematical model (an algorithm) defining a placement of a section having gas accidental escape in complex main gas pipeline with several sections and branches is suggested. The algorithm does not required knowledge of corresponding initial hydraulic parameters at entrance and ending points of each sections of pipeline (receiving of this information is rather difficult without using telemetric informational system). The algorithm is based on mathematical model describing gas stationary movement in the simple gas pipeline and upon some results followed from that analytical solution and computing calculations.

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Математическое Моделирование Неординарного Течения Газа в Главном Газопроводе/Т.Давиташвили, И.Самхарадзе Г. Губелидзе/.Сб. Трудов Института Гидрометеорологии Грузинского Технического Университета Грузии. –2011. – т.117. – с. 162-165. – Анг .; Рез. Груз., Анг.,Рус.

В этой статье предлогаеться математическая модель (алгоритм) для определения местонахождения секции, имеющей утечку газа в сложном главном газопроводе с несколькими секциями и ветвями. Алгоритм не требует знания соответствующих начальных гидравлических параметров в начальных и в конечных пунктах каждой секции трубопровода (получение этой информации является довольно сложным, без использования телеметрической информационной системы). Алгоритм основан, на математической модели описывающей стационарнное движение газа в сложном газопроводе, на аналитическом решений уравнений описывающих стационарнное движение газа и на результатов некоторых вычислений.