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MUTUAL INFLUENCE OF THE ATMOSPHERE AND THE OCEAN UNDER WAVE PROCESSES

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Summary: *In the report the problem of surface gravitational waves using the theory of tangential discontinuity between media: air-water is considered. Using the improved equation of mass continuity and taking into account the atmosphere inhomogeneity in the gravitational field of the Earth, it is shown that during wave processes, these two media mutually influence each other, which explains the reason for the formation of a stormy condition over the ocean and the drop in atmospheric pressure before the storm. The mechanism of the formation of the "killer wave" has been established and thus the "greatest mystery of nature" has been solved. The scale of wind and tsunami wavelengths has been established.*

Key Words: *Atmosphere, Ocean, Gravitational Waves, Waves of Wind, Tsunami Waves, Killer Wave.*

Introduction. Surface gravity waves are the most common natural phenomena. These waves are divided into two types - wind waves and tsunami waves, however, there is no clear criterion that distinguishes the lengths of these waves. These waves are generated and propagated at the interface between water and air and, therefore, studying them is a typical task of tangential discontinuity. In modern theory, these waves are investigated using the equations of hydrodynamics. relating only to water, which excludes the possibility of establishing the influence of the atmosphere on the problem under study. In addition, it is assumed that water is incompressible and its motion is potential, which leads to many contradictions in the problems of hydrodynamics and thus is not true [1,2] .

We obtain the dispersive equations of gravitational waves for an inhomogeneous atmosphere and water using the improved Euler and mass continuity equations and then, in accordance with the theory of tangential discontinuity, we stitch their solutions at the interface using the correct boundary conditions. This approach makes it possible to reveal the mutual influence of the ocean and the atmosphere during wave processes, to establish the causes of origin of the storm and "killer wave" and also to establish the scale of the wavelengths of the wind and tsunami.

Methods. The system of improved equations of hydrodynamics has the form [3]

$$\begin{cases} \rho \dfrac{\partial \vec{V}}{\partial t} = -\nabla P + \rho \vec{g} \\ \dfrac{\partial \rho}{\partial t} + \left(\vec{V}\nabla\right)\rho = -\rho \nabla \vec{V} - \dfrac{\vec{V}\nabla P}{C_p^2} \end{cases}, \tag{1}$$

Where $C_p -$ isobaric speed of sound, which determines the degree of inhomogeneity of the medium, In a homogeneous medium $C_p = \infty$ [4]. Linearizing system (1) with using the equation of state of the medium $\rho' = P'/C^2$ and the equilibrium condition in the gravitational field of the Earth $\nabla P_0 = \vec{g}\rho_0$, where P_0, ρ_0 and P', ρ' stationary and perturbed values of pressure and density and also representing all perturbed quantities in the form of a plane wave $f'(x,z,t) = f_a(z)\exp i(kx - \omega t)$, where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength and ω is its frequency, we obtain the equation of a gravitational wave with respect to the amplitude of perturbed pressure in the form:

$$\frac{d^2P_a(z)}{dz^2}+\frac{g}{C_s^2(z)}\frac{dP_a(z)}{dz}-\left(k^2+\Omega\ (z)-\frac{\omega^2}{C^2(z)}\right)P_a(z)=0 \quad , \qquad (2)$$

Here C is the true value of the speed of sound in atmosphere, which is related to the adiabatic C_s and isobaric C_p speeds of sound by the ratio $C^2=C_s^2C_p^2/\left(C_s^2+C_p^2\right)$. $\Omega(z)$ is a value that depends on C, C_s, C_p and their derivatives along the vertical coordinate z. The atmosphere is an inhomogeneous medium due to the influence of the Earth's gravitational field on it, and water is a homogeneous medium, since the gravitational force is negligible in comparison with intermolecular forces. Therefore, equation (2) for air is a second order differential equation with variable coefficients, the analytical solution of which is impossible. Based on this, we average these coefficients in the troposphere in the height interval from $z=0$ to $z=10^4 m$, where their dependences on z are known

$$\overline{C}_{s1}=320m/\sec, \overline{C}_{p1}=590m/\sec, \overline{C}_1=280m/\sec, \overline{\Omega}_1=-8.19\times10^{-9}m^{-2} \qquad (3)$$

For water, these coefficients do not depend on z and therefore

$$C_{p2}=\infty, C_{s2}=C_2=\overline{C}_2=1480m/sek \text{ и } \Omega_2=\overline{\Omega}_2=0 \qquad (4)$$

We will seek a solution to equation (2) for air in the form $P_{a1}(z)=A\exp(\gamma z)$ and for water $P_{a2}(z)=B\exp(\delta z)$, after which they will have the form:

$$P_{a1}(z)=A_1\exp(\gamma_1 z)+A_2\exp(\gamma_2 z) \quad , \qquad P_{a2}(z)=B_1\exp(\delta_1 z)+B_2\exp(\delta_2 z) \quad ,$$

(5)

where A_1, A_2, B_1, B_2 are constants and

$$\gamma_1=-\frac{k}{\theta_{s1}}\left[1+\sqrt{1+\theta_{s1}^2\left(1+\frac{\overline{\Omega}_1}{k^2}-x^2\right)}\right],\ \gamma_2=-\frac{k}{\theta_{s1}}\left[1-\sqrt{1+\theta_{s1}^2\left(1+\frac{\overline{\Omega}_1}{k^2}-x^2\right)}\right] \qquad (6)$$

$$\delta_1=-\frac{k}{\theta_{s2}}\left[1+\sqrt{1+\theta_{s2}^2\left(1-\frac{C_1^2}{C_2^2}x^2\right)}\right]<0,\ \delta_2=-\frac{k}{\theta_{s2}}\left[1-\sqrt{1+\theta_{s2}^2\left(1-\frac{C_1^2}{C_2^2}x^2\right)}\right]>0 \qquad (7)$$

$$\theta_{s1}=\frac{2k\overline{C}_{s1}^2}{g},\ \theta_{s2}=\frac{2kC_2^2}{g},\quad x=\frac{U_p}{C_1},\ U_p=\frac{\omega}{k}$$.- phase velocity of wave. (8)

Since the wave is surface and the atmosphere is not bounded from above, the condition $P_{a1}(z)\rightarrow 0$ at $z\rightarrow\infty$ must be fulfilled. It is seen from (6) that if fulfilled the condition

$$1+\theta_{s1}^2\left(1+\frac{\overline{\Omega}_1}{k^2}-x^2\right)>1$$

(9) $\gamma_1<0$ and $\gamma_2>0$ and then only the first term remains in the expression for $P_{a1}(z)$. If

$$0\le 1+\theta_{s1}^2\left(1+\frac{\overline{\Omega}_1}{k^2}-x^2\right)<1 \qquad (10)$$

$\gamma_1<0, \gamma_2<0$ and both terms are present in. Substituting the value $\overline{\Omega}_1$ and parameter values from (3), solutions (9) and (10) respectively, will be:

$$k>9{,}05\times10^{-5}/\sqrt{1-x^2}m^{-1}\Rightarrow\lambda<6.94\times10^4\sqrt{1-x^2}m \qquad (11)$$

$$k < 9{,}05 \times 10^{-5} / \sqrt{1-x^2}\, m^{-1} \Rightarrow \lambda > 6{,}94 \times 10^4 \sqrt{1-x^2}\, m \tag{12}$$

Waves whose lengths satisfy condition (11) will be called wind waves, and condition (12) - tsunami waves. The boundary conditions used by solving the problem are as follows:

$$P_2\big|_{z=0} = P_1\big|_{z=0} + \rho_{02} g \varsigma(x,t), \quad V_{z1}\big|_{z=0} = V_{z2}\big|_{z=0} = \frac{\partial \xi}{\partial t}, \quad V_{z2}\big|_{z=-H} = 0, \tag{13}$$

where $\xi(x,t) = a \exp[i(kx - \omega t)]$ is the vertical displacement of the surface of the tangential discontinuity and $H-$ the depth of the ocean. These conditions give the following dispersion equations for waves of wind and tsunami respectively:

$$\left\{ \frac{2}{\theta_{p1}} + \frac{1}{\vartheta_{s1}} \left[1 - \sqrt{1 + \theta_{s1}^2 \left(1 + \frac{\overline{\Omega}_1}{k^2} - x \right)} \right] \right\} \times \left[\frac{kg}{\omega^2} \tanh(kH) - 1 \right] = 0$$

(14)

$$\left\{ \frac{2}{\theta_{p1}} + \frac{1}{\vartheta_{s1}} \left[1 + \sqrt{1 + \theta_{s1}^2 (1-x)} \right] \right\} \times \left[\left(\frac{kg}{\omega^2} + \frac{1}{\theta_{s2}} \right) \tanh(kH) - 1 \right] = 0$$

(15)

Results and discussion. Equation (14) splits into two equations:

$$\frac{2}{\theta_{p1}} + \frac{1}{\theta_{s1}} \left[1 - \sqrt{1 + \theta_{s1}^2 \left(1 + \frac{\overline{\Omega}_1}{k^2} - x^2 \right)} \right] = 0 \tag{16}$$

$$\frac{kg}{\omega^2} \tanh(kH) - 1 = 0 \tag{17}$$

Equation (16) describes longitudinal waves in the air excited by perturbationson the surface of water and propagating along this surface. Substituting the values of the parameters from (3), its solution will be:

$$|x| = \sqrt{1 - \frac{1{,}17 \times 10^{-8}}{k^2}} \Rightarrow |U_p| = \sqrt{1 - \frac{1{,}17 \times 10^{-8}}{k^2}}\, \overline{C}_1 = \sqrt{1 - \frac{1{,}17 \times 10^{-8} \lambda^2}{4\pi^2}}\, \overline{C}_1 \tag{18}$$

From (18) we find

$$k = \frac{1{,}08 \times 10^{-4}}{\sqrt{1-x^2}} \tag{19}$$

which is consistent with the condition (11) and therefore the roots (18) are not extraneous for any valid values *k*. We see that for $k > 10^{-3} m^{-1} \Rightarrow \lambda < 6{,}3 \times 10^3 m$, the phase velocity of the wave in the air is constant and equal to $U_p = \overline{C}_1 \cong 280 m/\sec$, and then, witha decrease of k, it falls and at $k = 1.08 \times 10^{-4} m^{-1} \Rightarrow \lambda = 5.81 \times 10^4 m$ we have $x = 0$ i.e. , the wave stops. This means that at this wavelength, two regions of about 30 km long, with high and low pressures are formed in the atmosphere above water. In the area of low atmospheric pressure, the amplitude of the surface wave will increase and the pressure difference between the two areas will lead to the appearance of wind. When is further reduced k to its minimum value, which is defined from (11) $k_{\min} \cong 9 \times 10^{-5} m^{-1} \Rightarrow \lambda_{\max} \cong 7 \times 10^4 m$, the roots of equation (18) and the corresponding frequencies of standing waves in the atmosphere become imaginary, which leads to a sharp increase in the pressure difference and, consequently, the amplitude of the surface wave and the wind force. This result clearly explains the reason for the drop in atmospheric pressure over the sea and ocean before the storm, as well as the reason for its strengthening..

The solution to equation (17), which describes surface gravitational waves on water, is:

$$\left|U_p\right| = \sqrt{\frac{g}{k}\tanh(kH)} = \sqrt{\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi H}{\lambda}\right)} \tag{20}$$

which coincides with the well-known expression for the phase velocities of wind and tsunami waves.

The phase velocity of a wave in the atmosphere does not depend on the depth of the ocean and decreases from the speed of sound to zero with increasing wavelength. In the ocean, on the contrary, the phase velocity increases from zero with increasing wavelength and depth. It is evident, that at certain wavelengths, which will depend on the depth of the ocean, these speeds will coincide and resonance of the frequencies will occur of waves in the atmosphere and the ocean. This fact is shown in Fig. 1.

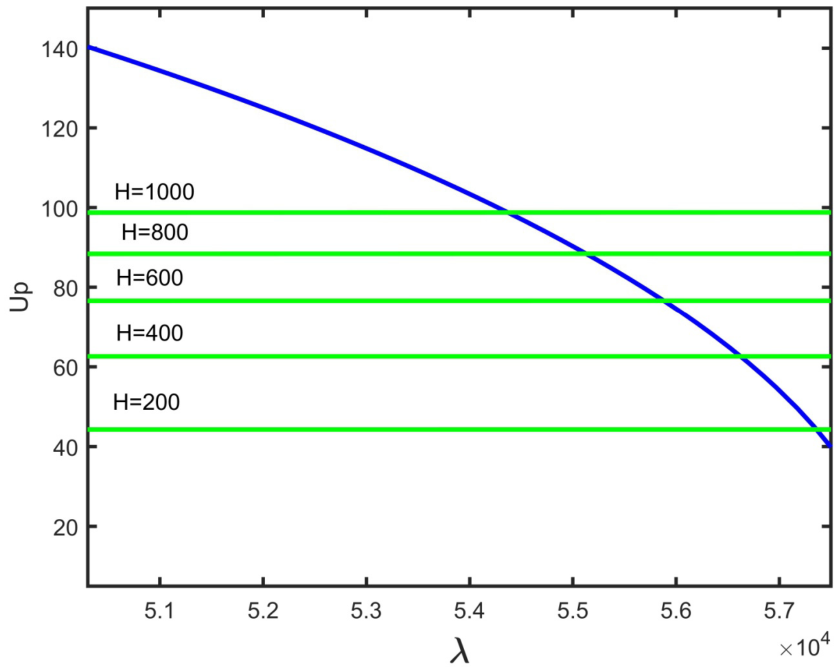

Fig. 1. Plots of dependences of the phase velocities of longitudinal waves on the air (18) (blue curve) and on the water surface (20) (green curves) for different depths.

We can assume that this resonance is the cause of origin of the “killer wave”, especially since there is no other explanation yet. It should be noted, that the resonance alone is not enough for the appearance of a “killer wave”—it is essential that the oscillations in the air and in the water are occurring in antiphase.

Equation (15) for tsunami waves also splits into two equations, the first of which has no solution based on the condition (10), which means that tsunami waves do not affect the atmosphere. The solution to the second equation coincides with (20).

Conclusion. This report does not claim to be highly accurate or to be the ultimate truth. The upper boundary of the troposphere changes depending on geographic parameters, and this concludes, that the average values of the problem parameters calculated here and, therefore, all the numerical data given in him are rather conditional. However, undoubtedly the proposed method for solving the problem is new and makes it possible to trace the correlation between the ocean and the atmosphere during wave processes. In particular, it became clear why the atmospheric pressure in the ocean drops before the storm, as well as differentiating between the wavelengths of wind and tsunami became possible. Its apparent advantage is also that at the level of a highly plausible hypothesis, it reveals the greatest mystery of nature called the “Killer Wave”. Now it is clear. why this wave is solitary. This is due to the fact that the flat relief of the ocean floor is disturbed at the distances of the order of the wavelengths that we calculated. It is also clear why a cavity, is formed before the wave. This is attributed to the fact that there is a region in front of the wave, where the pressure in the water sharply drops and in the atmosphere sharply increases.

Acknowledgement. For a detailed discussion of the problem, see the work published on the site: https://www.scirp.org/journal/paperinformation.aspx?paperid=110796

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