TEMPORAL ANALYSIS OF EARTHQUAKES AND APPROXIMATION BY QUANTILE FUNCTION

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Abstract: *In this research the new approach of estimation and approximation of quantile and quantile density function when we know the moments were investigated. In the role of moments, we use time intervals (waiting times) between earthquakes of the seismic catalog. Data on waiting times between earthquakes taken from the worldwide seismic catalogs were investigated. We will analyze three models of approximation and estimation: frequency moments, usual moments, and transposition moments. There are various models of quantile function estimation but the advantage of our model is that it is enough to have moments.*

Key Words: *Earthquake, waiting time, quantile function*

Introduction. Among the results of these studies, we can name the well-known findings of temporal properties of seismicity, based on modern views on the fractal nature of tectonics, faults, and hypocenter distributions. It is in full agreement with the above view of the complex dynamic nature of the seismic process and argues that the dynamics of earthquake time are characterized by "switching" or "shifting", which is manifested by alternating periods of increased seismic activity and low seismic activity. Nevertheless, the main aspects of the problem of the time distribution of intervals between earthquakes are still unclear and sometimes the study of the issue has not been touched yet [1-3].

Nowadays the moment problem is one of the most important problems in statistics and is used in mathematics, financial mathematics, economics, insurance, etc. During the last three centuries the moment problem has been studied in various articles, but still today it is a very interesting problem in mathematics [4-6].

In this research, we will discuss the approximation and estimation of quantile function, in the case when we have the moments [7].

We can say that the moment problem has the only solution, when the system of equations $\int x^j dF(x) = \int x^j dG(x) \quad j = 0,1\ldots$, has one solution, $F = G$.

Methods. There are various nonparametric methods of estimation of quantile function. For example, Harrell, and Davis [8] discuss statistical sums. Bolance et al., [9] and Brewer [10] in their work discuss the Bernstein polynomial quantile function estimation.

As for the new concept we will use in this research is that it has an advantage and can be used in the case when we have less information about the distribution function, for example, the values of moments.

We will combine the investigation of time intervals (waiting times) between earthquakes and approximation of the results by quantile function using modern processing methods. Modern methods of time distribution research based on traditional linear and nonlinear concepts will be used. By the analysis of waiting times series of different origins, we plan to unravel features of earthquakes' time evolution.

The project is in full compliance with the main problem of the Earth sciences. We will combine modern data analysis tools in the specially developed user software package.

For analysis, three models of approximation and estimation have been chosen. The first model we use when we have so-called frequency moments. The second model is about usual moments and the third model we use when we have transposition moments.

Results and discussion. For analysis of time interval (waiting times) of California seismic catalog. The data discussed are taken from the Southern California Local Earthquake Catalog, which can be viewed at the site (http://www.data.scec.org/ftp/catalogs/). Analysis for the 1932-2013 period was analyzed.

This catalog is more reliable because the collection of data was almost non-stop during this period.

After that, by using programs created on the base of Ivane Javakhishvili Tbilisi State University M. Nodia Institute of Geophysics the time intervals (waiting times) between earthquake data sets were carried out.

Using waiting times data the approximate estimate of the quantile function of the first model was calculated and will compared with the famous Harrell Davis estimate. For analysis, we use the following formulas:

$$\hat{Q}_{HD}(x) = \sum_{i=1}^{n} X_{(i)} \int_{\frac{i-1}{n}}^{\frac{i}{n}} \beta(y, \lfloor \alpha x \rfloor + 1, \alpha - \lfloor \alpha x \rfloor + 1) dy$$

$$\hat{Q}_{\alpha}^{-}(x) = \sum_{i=1}^{n+1} \Delta X_{(i)} B_{\alpha}\left(\frac{i-1}{n}, x\right) = \sum_{i=1}^{n} X_{(i)} \left[B_{\alpha}\left(\frac{i-1}{n}, x\right) - B_{\alpha}\left(\frac{i}{n}, x\right) \right]$$

The behavior for different parameters for first model will be as follows (Fig.1- Fig.3).

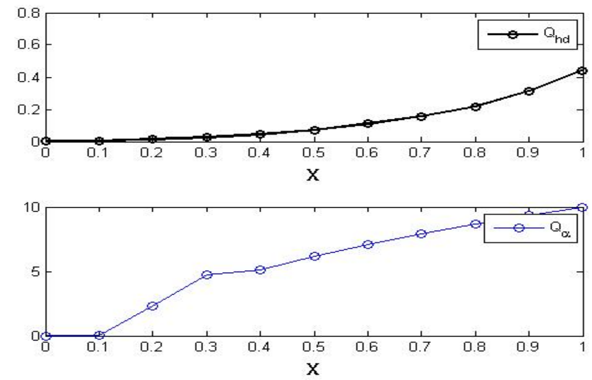

Fig. 1. Comparison of estimates of Harrell Davis and the first model (frequency moments) for the waiting time when. $\alpha = 20, n = 100$

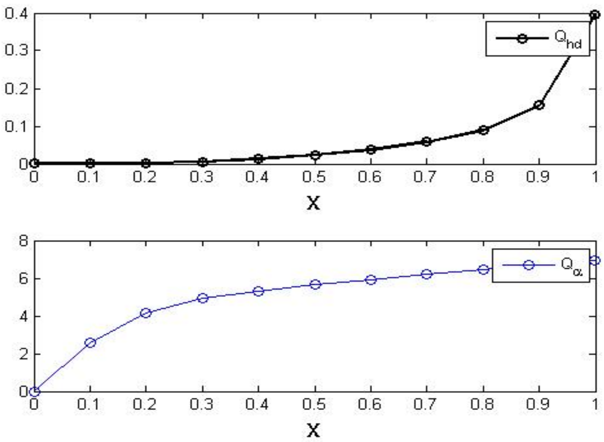

Fig. 2. Comparison of the estimates of Harrell Davis and the first model (frequency moments), for the waiting time when $\alpha = 50, n = 100$.

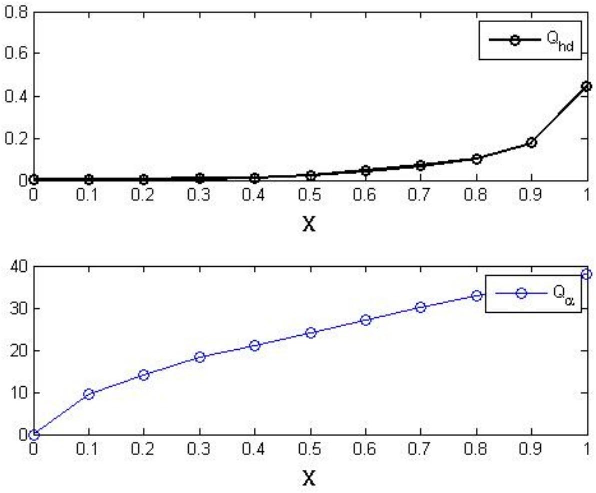

Fig. 3. Comparison of the estimates of Harrell Davis and the first model (frequency moments), for the waiting time when $\alpha = 100, n = 100$.

The results of the research will help to understand the nature of seismic processes and what most important it will contribute in the future to solve the problem of understanding the nature of strong earthquakes.

Conclusion. Our research presents the analysis of time distribution between earthquake events and uses the following datasets for approximation and estimation of quantile function. The approximation of quantile function can be used in different fields: financial mathematics, economics, insurance, etc. The research of the project is global, it includes various important tasks from different scientific fields. The research has both fundamental and practical importance because without understanding the temporal distribution of earthquakes, we cannot estimate the hazard and make serious progress in the task of understanding the nature of earthquakes.

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References

[1] Telesca L., Matcharashvili T., Chelidze T. Investigation of the temporal fluctuations of the 1960–2010 seismicity of Caucasus. // Nat. Hazards Earth Syst. Sci., Göttingen Germany, 12, 2012, pp. 1905–1909.

[2] Talbi A., Yamazaki F. Sensitivity analysis of the parameters of earthquake recurrence time power law scaling. // J. Seismol, Springer, Netherlands, 13, 2009, pp. 53–72.

[3] Molchan G. Interevent time distribution in seismicity: a theoretical approach. // Pure Appl Geophys., Springer, Netherlands, 162, 2005, pp. 1135–1150.

[4] Mnatsakanov R., Sborshchikovi A. Recovery of a Quantile Function from Moments. // Journal of Computational and Applied Mathematics, Elsevier, Netherlands, 315, 2017, pp. 354-364.

[5] Mnatsakanov R., Sborshchikovi A. Recovery of quantile and quantile density function using the frequency moments. // Journal of Statistics and Probability Letters, Elsevier, Netherlands, 140, 2018, pp. 53-62.

[6] Sborshchikovi A. On Nonparametric Quantile Function Estimation Using Transformed Moments. // Bulletin of the Georgian National Academy of Sciences, Tbilisi, Georgia, 11(3), 2017, pp. 22-27.

[7] Cheng C., Parzen E. Unified estimators of smooth quantile and quantile density functions. // J. Statist. Plann. Inference, Elsevier, Netherlands, 59, 1997, pp. 291–307.

[8] Harrell F.E., Davis C.E. A new distribution-free quantile estimator. // Biometrika, Oxford University Press, 69, 1982, pp. 635–640.

[9] Bolance C., Guillen M., Nielsen, J. Transformation kernel estimation of insurance claim cost distributions. // in: M. Corazza, et al. (Eds.), Mathematical and Statistical Methods for Actuarial Sciences and Finance, Springer-Verlag, Italy, 2010, pp. 43–51.

[10] Brewer K.R.W. Likelihood Based Estimation of Quantiles and Density Estimation, 1986.